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2014 Mathematical Contest in Modeling (MCM) Summary Sheet

(Attach a copy of this page to your solution paper.)

Type a summary of your results on this page. Do not include the name of your school, advisor, or team members on this page.

Keep-Right-Except-To-Pass rule is common in many countries but its performance has not been fully studied. In this article, we develop a hierarchy of microscopic and macroscopic models to analyze this rule. A road with no such lane-changing rule at all is taken as the reference group for comparison.

Our microscopic model analyzes various kinds of important interactions between two cars on the road. Our macroscopic model, based on the well-known gas-like kinetic equation, analyzes the interaction and evolution of the entire road from a view of continuous traffic flow. We combine two levels of models together by setting functions needed in the equation (macroscopic) to be results computed from a particle's point of view (microscopic).

We define two variables to measure the safety and efficiency of the road. One is accident coefficient which is defined to be the total chance of car accident. Another is phase-space density (PSD) which is defined to the density of a car with particular velocity and maximum velocity at a specific location and time. They are our goal variables.

Numerical simulation is our approach to solve the hierarchy model. Besides solving the major model, we have analyzed the impact of different factors including inhomogeneity of different lanes, low and heavy traffic, total number of lanes, under- and over-posted limits etc.

The numerical output gives us some remarkable results: The impact of the rule can be summarized as 'first use right, then use left' and 'slow cars use right, fast cars use left'. It promotes flow when traffic is heavy but decreases flow when traffic is light. Besides, it consistently keep the accident coefficient low. Safety and flow are generally negative correlated. A modest speed limit and car distance limit would promote overall road condition.

At last we propose two new alternative rules: Variable Speed Range and Car Distance Limits. Numerical tests prove their advantage. The possible effects of switched orientation and intelligent traffic system are also widely explored, although not strictly tested.

Evaluate A Lane-Changing Rule Based On Hierarchical Traffic Model

Feb 10th, 2014

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1 Introduction

This paper focuses on the analysis of the so call *Keep-Right-Except-To-Pass* rule, which is widely employed. Taking into account more factors including speed limits, we will analyze the traffic flow, safety coefficient and many other character of the road. By comparing a road under *Keep-Right-Except-To-Pass* rule and a road without *Keep-Right-Except-To-Pass* rule, we will analyze how the rule would affect the road traffic. Whether it would promote traffic flow is the central topic. At last, we would consider some more interesting factors like driving asymmetry and intelligent driving system to explore deeper into the issue.

1.1 Why Hierarchical Model?

Traffic is one of the central social and economical problems that has been studied for long. Many different models with different mathematical representation and different perspectives have been proposed,including Gas-like kinetic equation, cellular automata, LWR models etc. Traffic research has a history longer than 100 years. All these have indicated the complexity of this topic. Traffic involves many factors and many different situations which make it almost impossible to take all things into account in one model. Within the range of our interest, the biggest challenge is the interaction between cars which is of various types and high frequency. As the new trend of traffic research in recent years, we propose a hierarchy model to do our analysis.

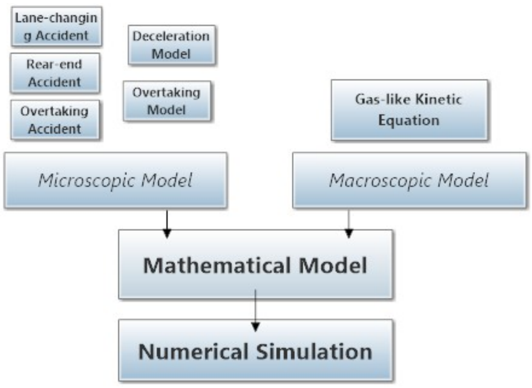

Figure 1: Hierarchy of models

As shown in the figure, the mathematical models consist of microscopic models and macroscopic models. The microscopic models focuses on the interaction between two cars while the macroscopic models analyze the behavior of the entire road.

1.2 Our General Approach

As our major goal is to analyze the traffic flow and traffic safety, we build 5 microscopic sub-models to analyze the following 5 issues. The first 3 focuses on traffic safety. The rest 2 focuses on traffic flow.

- **Lane-changing Accident:** the probability of an accident when changing the lane.

- **Overtaking Accident:** the probability of an accident when overtaking another car.
- **Rear-end Accident:** the probability of an rear-end accident when approaching the leading car.
- **Deceleration Model:** analyze how would a fast decelerate car when approaching a slower leading car.
- **Overtaking Model:** analyze the acceleration changes when a car passes another car on its right.

For the macroscopic model, the cornerstone is the **Gas-like kinetic equation**. First of all we transform discrete car numbers into continuous car density then build a partial differential equation. Our major focus is how to determine the amount of car density that changes lane. Based on several simple assumptions, we give the formulae of the lane-changing term mainly using integral operations. By similar techniques, the macroscopic safety coefficient is also determined.

Our strategy to combine the two kinds of models together is very similar to that of thermodynamics. The microscopic model is like molecular dynamics simulations and the macroscopic model is like statistical methods. The microscopic model gives the basic law of interaction and the macroscopic model estimates how the microscopic interaction effects the macroscopic behavior.

1.3 Notations

Table 1: Notation Table

Notation	Meaning
l_{notice}	the distance another overtaking car is behind a car when the driver notice it
t_1	average reaction time of a driver
t_{LC}	lane-changing time
α	careless factor in the overtaking accident model
$d_{overtake}$	dangerous rate in the overtaking accident model
$PICUD$	Potential Index for Collision with Urgent Deceleration
d_{LC1}, d_{LC2}	dangerous rate in the changing lane accident model
L_{safe}	safe distance
d_{rear}	dangerous rate in the Rear-End collision accident model
L_{dcc}	minimum decelerating distance in the deceleration model
d	deceleration rate in the deceleration model
a	acceleration
ρ_i	phase-space density on lane i
v_0	ideal velocity
$(\partial_t \rho_i)_{acc}$	phase-space density chance in time due to acceleration
$(\partial_t \rho_i)_{dcc}$	phase-space density change in time due to deceleration
$(\partial_t \rho_i)_{LC-} + (\partial_t \rho_i)_{LC+}$	phase-space density change in time due to lane change
T	period of traffic inflow
τ	expected accelerating time
$\tilde{\rho}_i(x,t)$	reduced car density of lane i at position x and time t
$\alpha(u,v)$	decelerating pressure from a car ahead with speed u to a following car with speed v
$\beta(u,v)$	the lane-changing pressure from a car ahead with speed u to a following car with speed v
$P_{accelerate}, P_{lanechange}$	total decelerating and lane-changing pressure
Pr	the probability of changing lane
$d^i_{overtake}$	accident coefficient for the overtaking in lane i
d^i_{LC}	accident coefficient for lane changing in lane i
d_{rear}	accident coefficient for acceleration on a same lane
G	(gap), required distance between consecutive vehicles on a same lane

2 Microscopic Model

Traffic Safety

Assumptions[1]

- The accident rate is negatively correlated with the length of time between the time when the driver being aware of a certain event (like overtaking, changing lane) and the time the event really happens.

[1] The assumptions below are also applied to the analysis of traffic flow and may not be repeated in the next section of analyzing traffic flow

- The accident rate is positively correlated with the car's velocity.
- All drivers are intending to avoid accidents and having a certain level of cautiousness.

2.1 Overtaking Accident

Assume a car with speed (average speed during overtaking process) v_2 overtakes a car with speed v_1 and $v_2 > v_1$. Since the accident happening during overtaking are mainly because the driver of the overtaken vehicle is not enough aware that a car is passing him, the length of time that he realized a car was to overtake matters. Suppose the driver would notice the overtaking car when it is l_{notice} from him, then he would notice the overtaking car $\frac{l_{notice}}{v_2-v_1}$ time before it overtakes. And we define the carelessness factor as $\alpha = e^{t_1 \frac{v_2-v_1}{l_{notice}}}$, where t_1 is the average reaction time of a driver. When $v_2 >> v_1$, it is largely possible that the driver did not notice a car is overtaking thus can be quite careless while when $v_2 \approx v_1$, the driver has a long time to be aware of the overtaking car before it overtakes thus would be very careful. At last, the faster the car, the more dangerous if any improper behavior is conducted. In summary, we can define the danger (accident) coefficient as:

$$d_{overtake} = \frac{(e^{t_1 \frac{v_2-v_1}{l_{notice}}} - 1)}{(e^{t_1 \frac{v_{max}}{l_{notice}}} - 1)} \cdot \frac{v_1}{v_{max}} \tag{1}$$

2.2 Changing Lane Accident

From [3], we define the PICUD (Potential Index for Collision with Urgent Deceleration) as the distance difference between the two vehicles when they completely stop after the vehicle at the front suddenly applies emergency brake(which is followed by the emergency brake of the vehicle on the back). We may assume that the vehicle changing its lane (denoted by CL car) enjoys a speed v, where the car at the front is of speed v_1, with initial distance l_1, the car following the CL car has speed v_2 with distance l_2. The driver has reaction time t_1. By elementary calculation, we know that

$$PICUD_1 = l_1 - (v \cdot t_1 + \frac{v^2}{2d_{max}}) + \frac{v_1^2}{2d_{max}} \tag{2}$$

$$PICUD_2 = l_2 - (v_2 \cdot t_1 + \frac{v_2^2}{2d_{max}}) + \frac{v^2}{2d_{max}} \tag{3}$$

We define the probability of crashing by

$$d_{LC1}(v, v_1, l_1) = 1 - \max\{\frac{PICUD_1}{l_1}, 0\} \tag{4}$$

$$d_{LC2}(v, v_2, l_2) = 1 - \max\{\frac{PICUD_2}{l_2}, 0\} \tag{5}$$

2.3 Rear-end Accident

We can measure the safety of a moving car on the highway by the speed of itself, the speed of the car ahead, and the distance between them. Assume that the speed of the car ahead and the car behind are, respectively, v_0 and $v_1 (v_0 > v_1)$, and the distance between them is L. Assume the maximum deceleration a car can undergo when it brakes is d_{max}

($d_{max} > 0$), and a driver has an average reaction time t_1, as defined before. Assume the car ahead has an uniform speed.
The distance decrease between the two car in the reaction time is $L_1 = (v_0 - v_1) \cdot t_1$. The minimum time the rear car should spend slowing down to the speed of the front car is $t_{min} = \frac{(v_0 - v_1)}{d_{max}}$. Thus the safe distance of two cars are

$$L_{safe} = L_1 + \frac{t_{min} \times (v_0 - v_1)}{2} = (v_0 - v_1) \cdot t_1 + \frac{(v_0 - v_1)^2}{2d_{max}} \tag{6}$$

We can define a parameter d_{rear} to measure the safety, $d_{rear} \in [0, 1]$. $d_{rear} = 1$ is the most dangerous situation while $d_{rear} \to 0$ is the safest situation.

$$d_{rear}(v_0, v_1, L) = \frac{L_{safe}}{L} \tag{7}$$

Traffic Flow

Assumptions

- A car has a fixed maximum velocity.
- A car would always like to accelerate to its maximum speed if there is enough space and no slower cars near ahead.
- A car would always like to change lane to overtake when encountering a slower car ahead as long as there is space for changing lane on the left.
- A car should have zero acceleration/deceleration when passing another car, for safety concerns.
- For simplicity, we assume that acceleration only changes continuously & linearly with respect to time.
- A driver can tell, approximately, the position and velocity of cars near him.

2.4 Deceleration Model

We've assumed that a car would always accelerate to its maximum speed as long as there's acceleration space. Now we consider when would a car decelerate. As assumed, a car would always choose to change lane and overtake the ahead slower car if the overtaking lane is available. But when the left lane for overtaking is very crowded, the car cannot change lane to overtake. Instead, it would have to decelerate to follow the slower car. Now we analyze this process.
Suppose the current car is at speed v_0 and the ahead car is at speed v_1, where $v_1 < v_0$. The two cars have a distance l. We've already got the formula for safety distance when discussing the rear-end accidents.

$$L_{safe} = (v_0 - v_1) \cdot t_1 + \frac{(v_0 - v_1)^2}{2d_{max}} \tag{8}$$

Clearly, if the faster car start decelerating at safety distance, it is already very dangerous. We define minimum decelerating distance as the distance a car would run when decelerate at its maximum decelerating rate to the speed of the leading slower car.

$$L_{dcc} = \frac{d_{max}}{2} \cdot (\frac{(v_0 - v_1)}{d_{max}})^2 + (v_0 - v_1) \cdot \frac{(v_0 - v_1)}{d_{max}} \tag{9}$$

Then clearly $L_{dcc} + L_{safe}$ is the minimum distance to start deceleration safely. Surely most drivers are more cautious. We assume a driver would start to decelerate at distance $2L_{dcc} + L_{safe}$. The deceleration rate is determined by the following rule: if the car keep the current decelerating rate, it would become relative static with the leading slower car exactly at the safety distance. Therefore deceleration rate a is determined by the following equation.

$$\frac{d}{2} \cdot (\frac{(v_0 - v_1)}{d})^2 + (v_0 - v_1) \cdot \frac{(v_0 - v_1)}{d} = l - L_{safe} \tag{10}$$

Solve this equation we finally obtain the formula of deceleration rate. Note that the deceleration rate will never exceed the maximum deceleration rate.

$$d(v_0, v_1, l, t) = \begin{cases} min\{\frac{3(v_0-v_1)^2}{2(l-L_{safe})}, d_{max}\} & \text{if} \quad l < 2L_{dcc} + L_{safe} \\ 0 & \text{if} \quad l \geq 2L_{dcc} + L_{safe} \end{cases} \tag{11}$$

2.5 Overtaking Model

Now we analyze the case under which a car changes lane, overtakes and goes back the original lane.Assume the following car has speed v_0 and the leading car has speed v_1. After the overtaking, the following car goes back to the original lane with speed $v_0{}'$.

First, we have to know where the following car would change lane to the left. We have already had careful discussion about safety distance in previous parts and we can directly apply the results. We assume that a car would change lane to overtake at the distance $L_{safe}(v_0, v_1)$ from the leading car. When it has changed the lane, it would increase its acceleration to pass the car then decrease its acceleration to change back to the previous lane. Moreover, the position where the car turns back is $L_{safe}(v_1, v_0{}')$ distance ahead of the original leading car. Based on the assumptions, we can derive the following equations.

$$\int_0^{t_0} \int_0^{t_0} a(t)dtdt + (v_0 - v_1) \cdot t_0 = L_{safe}(v_0, v_1) \tag{12}$$

$$\int_0^{t_1} \int_0^{t_1} a(t)dtdt + (v_0 - v_1) \cdot t_1 = L_{safe}(v_0, v_1) + L_{safe}(v_1, v_0 + \int_0^{t_1} a(t)dt) \tag{13}$$

$$a(0) = a(t_1) = 0 \tag{14}$$

$$a(t_0) = a_{max} \tag{15}$$

In the equation, t_0 is the time when overtaking happens and t_1 is the time the car turns back to its original lane.

It is a complex integral equation. With our assumption of linear increase and decrease of a, one can easily verify that the equations are algebraically solvable but highly complicated. So we relax our condition that the car does not need to return the original line exactly at $L_{safe}(v_1, v_0{}')$ ahead. Instead, as long as the distance is larger than the safety distance, returning back would be okay. So the car would decrease its acceleration at the same rate as it increased acceleration. And when the acceleration is decreased to zero and it still within the safety distance, it would drive at constant speed until it reaches the safety distance and then turns back to the right lane. Finally, we can give the explicit formula of acceleration during an overtaking.

$$t_0 = \frac{\sqrt{9(v_0 - v)^2 + 6L_{safe}(v_0, v_1)a_{max}} - 3(v_0 - v)}{a_{max}} \tag{16}$$

$$k = \frac{a_{max}^2}{\sqrt{9(v_0 - v)^2 + 6L_{safe}(v_0, v_1)a_{max}} - 3(v_0 - v)} \tag{17}$$

$$a(v_0, v_1, t) = \begin{cases} k \cdot t & \text{if} \quad 0 \leq t \leq t_0 \\ k \cdot t_0 - k \cdot (t - t_0) & \text{if} \quad t_0 < t \leq 2t_0 \end{cases} \tag{18}$$

Note that in the formula, $t = 0$ represents the time when the following car is just at the safety distance from the leading car and changes its lane.

3 Macroscopic Model

As we have already built microscopic model to analyze the iteration between two cars, we proceed to consider the traffic behavior of an entire road. Our goal function is defined as follows.

Definition 1 $\rho_i(v, v_0, x, t)$ = *The phase-space density (PSD) of vehicles with velocity* v*, maximum velocity* v_0 *at position* x *and time* t *on road* i*.*

Assumptions

- All cars with different velocities would spend the same amount of time t_{LC} changing a lane.
- The length of a vehicle is neglected.
- A car would slow down only if there are slow cars ahead and there is no space on the left lane for overtaking.
- The maximum velocities of the vehicles are normally distributed. This is empirically justified in [2]
- The inflow of the road is periodically vibrating.
- *Keep-Right-Except-To-Pass Law* : A car would instantly return back to right lane after overtaking as long as there is space.
- Without *Keep-Right-Except-To-Pass Law*: A car would remain on its current lane only when having to change lane and overtaking. After overtaking, the car would stay on the overtaking lane until next time having to overtake. The car has equal probabilities to choose to overtake on the left lane and right lane.

Traffic Flow

$\rho(v, v_0, x, t)$ is the phase-space density of vehicle with velocity v, ideal velocity v_0 at position x and time t. We fist give the following definition.

Definition 2 $\tilde{\rho}_i(x, t) = \int_{v_{min}}^{v_{max}} \int_{v_{min}}^{v_{max}} \rho_i(v, v_0, x, t) dv_0 dv$

$\tilde{\rho}_i(x, t)$ is the reduced car density of lane i at position x and time t.
The following gas-kinetic like traffic equation[1] describes the macroscopic behavior of vehicle density on the road.

$$\partial_t \rho_i + v \partial_x \rho_i = (\partial_t \rho_i)_{acc} + (\partial_t \rho_i)_{dcc} + (\partial_t \rho_i)_{LC-} + (\partial_t \rho_i)_{LC+} \tag{19}$$

This equation means that the density change is equal to the sum of changes causes by acceleration, deceleration and lane-changing.
At the begin, the road is empty with no cars on it but cars start to enter the road at the entrance with starting velocities zero. As assumed, the maximum velocities satisfying the normal distribution and the inflow is periodically vibrating. So we can write the initial and boundary condition of the equation as:

$$\rho_i(v, v_0, 0, t) = 0 \quad \forall v > 0 \tag{20}$$

$$\rho_i(v, v_0, x, 0) = 0 \quad \forall x > 0 \tag{21}$$

$$\rho_i(0, v_0, 0, t) \sim N(\theta, \sigma^2) \cdot \sqrt{2} sin(\frac{2\pi}{T} \cdot t) \tag{22}$$

Then we consider the three terms on the right-hand side of the equation.
The first term $(\partial_t \rho_i)_{acc}$ represents the change of density due to acceleration. The following formula is given by [4] and is widely accepted. The term τ in the following equation is the accelerating time. It is a variable depending on the car density which measures the availability of accelerating space. Higher density, longer time it takes to accelerate to maximum speed. Therefore, $\frac{v_0 - v}{\tau}$ can be regarded as the expected acceleration.

$$(\partial_t \rho_i)_{acc} = -\partial_v(\rho_i \frac{v_0 - v}{\tau}) \tag{23}$$

Previous papers did not give an explicit way to calculate $\frac{v_0 - v}{\tau}$. Here we propose a precise formula to calculate $\frac{v_0 - v}{\tau}$. Let a_{max} be the maximum acceleration rate of the vehicle. We can get the following modified formula of $(\partial_t \rho_i)_{acc}$.

$$(\partial_t \rho_i)_{acc} = -\partial_v(\rho_i \frac{v_0 - v}{\tau}) = -\partial_v\{\rho_i a_{max} \cdot \frac{\int_v^{v_{max}} \int_{v_{min}}^{v_{max}} \rho_i(u, v_0, x, t) dv_0 du}{\tilde{\rho}_i(x, t)} e^{-\tilde{\rho}_i(x,t)}\} \tag{24}$$

This formula is intuitively reasonable. The numerator represents the total density of cars faster and the denominator represents the total density of cars at all velocities. This ratio can be regarded as the proportion of free space for acceleration because the cars running faster than you would not disturb your acceleration process.Acceleration is proportional to the free space ratio for acceleration. However, it does not only depend on the proportion of free space but also has a negative correlation with the total density. Since if there is a very high car density, even if most of the cars run faster then you, you still can not have a very big acceleration rate.
For the second term on the right-hand side,$(\partial_t \rho_i)_{dcc}$ represents the effect of deceleration. If there's a vehicle ahead of you that drives slower, you will have to decelerate or change lane to overtake. We can regard this as a wave propagating into an obstacle and receive a pressure, which can either be decelerating or lane-changing. We give the following two definitions:

Definition 3 $\alpha(u, v)$= *the decelerating pressure from a car ahead with speed* u *to a following car with speed* v *where* $u < v$.

Definition 4 $\beta(u, v)$= *the lane-changing pressure from a car ahead with speed* u *to a following car with speed* v *where* $u < v$.

The pressure term can be regarded as the **the loss of driving distance due to decelerating or lane-changing**.
For decelerating, the loss of driving distance is given by

$$\alpha(u,v) = \frac{1}{2} \cdot d_{max} \cdot (\frac{v-u}{d_{max}})^2 + (v-u) \cdot \frac{v-u}{d_{max}} = \frac{3}{2} \cdot \frac{(v-u)^2}{d_{max}} \tag{25}$$

For the lane-changing pressure, recall our assumption that when a faster car encounters a slower car, it would always like to change lane to overtake. Thus how much slower the following car is makes no difference. When a car changes its lane, it then has **no speed at all** on the original lane. Therefore the loss of driving length should be its current speed multiple the lane-changing time t_{LC}, which is a constant.

$$\beta(u,v) = v \cdot t_{LC} \tag{26}$$

Now the total decelerating and lane-changing pressure a car with speed v receives can be represented as the following integral.

$$P_{accelerate} = \int_{v_{min}}^{v} \int_{v_{min}}^{v_{max}} \rho_i(u, v_0, x, t)\alpha(u,v) dv_0 du \tag{27}$$

$$P_{lanechange} = \int_{v_{min}}^{v} \int_{v_{min}}^{v_{max}} \rho_i(u, v_0, x, t)\beta(u,v) dv_0 du \tag{28}$$

However, as one car can only choose one of slowing down and changing lane, we have to further determine the probability or proportion of cars decelerating and changing lane. We have assumed that all cars are wanting to run at their ideal velocities thus they are not willing to slow down unless they have to. The reason why some faster cars have to slow down instead of overtaking to keep its speed is that the left lane to use to overtake may be crowded. There may not be enough space to change lane and overtake. Therefore, the probability of changing lane Pr can be determined as a function of the car density on its current lane and its left lane.

$$Pr = e^{-\frac{\tilde{\rho}_{(i+1)}}{\tilde{\rho}_i}} \tag{29}$$

Lane $\#i+1$ is the lane left to lane $\#i$. The formula says that when the left lane is less crowded then the current lane, more cars will choose to change lane. In extreme situations, when lane $\#i+1$ is infinitely more crowded than lane $\#i$, the ratio becomes infinity thus the probability of lane-changing is zero. On the contrary, when lane $\#i+1$ is infinitely more vacant, the ratio becomes zero then the probability of lane-changing is one.
Finally the second and third term on the right-hand side of the equation can be determined.

$$(\partial_t \rho_i)_{dcc} = -\rho_i \cdot (1 - Pr) \cdot P_{accelerate} = -\rho_i \cdot (1 - e^{-\frac{\tilde{\rho}_{(i+1)}}{\tilde{\rho}_i}}) \int_{v_{min}}^{v} \int_{v_{min}}^{v_{max}} \rho_i(u, v_0, x, t)\alpha(u,v) dv_0 du \tag{30}$$

$$(\partial_t \rho_i)_{LC-} = -\rho_i \cdot Pr \cdot P_{lanechange} = -\rho_i \cdot e^{-\frac{\tilde{\rho}_{(i+1)}}{\tilde{\rho}_i}} \cdot \int_{v_{min}}^{v} \int_{v_{min}}^{v_{max}} \rho_i(u, v_0, x, t)\beta(u,v) dv_0 du \tag{31}$$

Now we have one last term $(\partial_t \rho_i)_{LC+}$ to be determined which measures the amount of cars entering this lane from other lanes. Using the same reduced density ratio argument, we directly gives:

$$(\partial_t \rho_i)_{LC+} = \rho_{i+1} \cdot Pr' \cdot P_{lanechange} = \rho_{i+1} \cdot e^{-\frac{\tilde{\rho}_i}{\tilde{\rho}_{(i+1)}}} \cdot \int_{v_{min}}^{v} \int_{v_{min}}^{v_{max}} \rho_i(u, v_0, x, t)\beta(u, v) dv_0 du \tag{32}$$

To get deeper into the formula, we can do a simple **dimensionality analysis**. The left-hand side term is taking partial derivative of ρ with respect to time t thus the dimensionality of the left term should be $\frac{1}{L \cdot V^2 \cdot T} \cdot \frac{1}{T}$. For the right-hand side term, the first term ρ has dimensionality $\frac{1}{L \cdot V^2 \cdot T}$ and the second term is a probability thus has dimensionality 1. The integrand has dimensionality $\frac{1}{L \cdot V^2 \cdot T} \cdot L$ thus the integral has dimensionality $\frac{1}{L \cdot V^2 \cdot T} \cdot T \cdot V^2$.

$$DIM_{left} = \frac{1}{L \cdot V^2 \cdot T} \cdot \frac{1}{T} = \frac{1}{L \cdot V^2 \cdot T^2} = \frac{1}{L \cdot V^2 \cdot T} \cdot \frac{1}{L \cdot V^2 \cdot T} \cdot L \cdot V^2 = DIM_{right} \tag{33}$$

Then we can see that the dimensionality of the left&right-hand side terms of the equation match. This also verifies our choice of the pressure term $\alpha(u, v)$ and $\beta(u, v)$ as the loss of driving distance with dimensionality L.

Note that the following two formulae only consider the right-most lane ($i.e. lane\#0$). For other lanes, lane-changing can be from or to two directions. With a little abuse of notations, we can obtain the new formulae.

$$\begin{aligned}(\partial_t \rho_i)_{LC-} = -\rho_i \cdot e^{-\frac{\tilde{\rho}_{(i+1)}}{\tilde{\rho}_i}} \cdot \int_{v_{min}}^{v} \int_{v_{min}}^{v_{max}} \rho_i(u, v_0, x, t)\beta(u, v) dv_0 du - \\ \rho_i \cdot e^{-\frac{\tilde{\rho}_{(i-1)}}{\tilde{\rho}_i}} \cdot \int_{v_{min}}^{v} \int_{v_{min}}^{v_{max}} \rho_{(i-1)}(u, v_0, x, t)\beta(u, v) dv_0 du\end{aligned} \tag{34}$$

$$\begin{aligned}(\partial_t \rho_i)_{LC+} = \rho_{(i+1)} \cdot e^{-\frac{\tilde{\rho}_i}{\tilde{\rho}_{(i+1)}}} \cdot \int_{v_{min}}^{v} \int_{v_{min}}^{v_{max}} \rho_i(u, v_0, x, t)\beta(u, v) dv_0 du + \\ \rho_{(i-1)} \cdot e^{-\frac{\tilde{\rho}_i}{\tilde{\rho}_{(i-1)}}} \cdot \int_{v_{min}}^{v} \int_{v_{min}}^{v_{max}} \rho_{(i-1)}(u, v_0, x, t)\beta(u, v) dv_0 du\end{aligned} \tag{35}$$

Note that by laws of reservation, the formula of $(\partial_t \rho_i)_{LC+}$ can be directly derived from that of $(\partial_t \rho_i)_{LC+}$.

Now we have derived a complete set of equations to model the behavior of road traffic under the *Keep-Right-Except-To-Pass* law. Then we consider the reference group. As assumed, in reference group a car will never change its lane unless having to overtake. First note that all other terms in the equation except the two lane-change ones would be the same. Second we assume that a car has the equal probability to choose to overtake on the right lane and the left lane.

The two figures show the difference more clearly. When the road is under the rule, a car would have two sources of pressure or impetus to change lane. One is to overtake while another is to return back to the right lane, following the rule. However, if the road is without the rule, there's only one pressure to change lane. That is to overtake. But this pressure is equally distributed to its left lane and right lane. Hence we can easily get

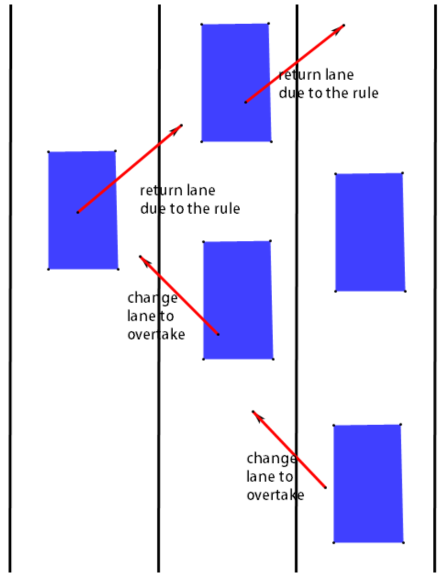

Figure 2: Traffic under the RULE

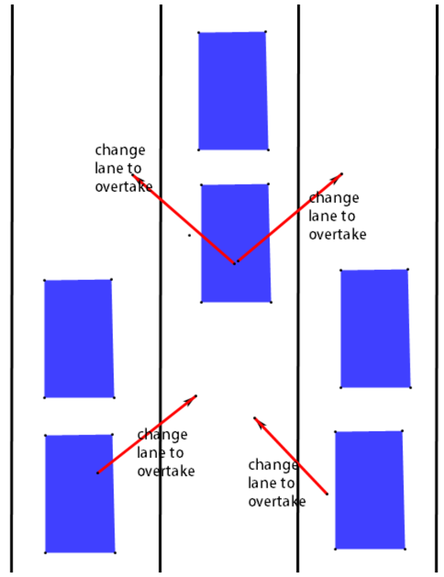

Figure 3: Traffic without the RULE

seemingly similar formulae but highly different in nature (check the subscripts carefully) for the lane-change terms.[2]

$$
\begin{aligned}
(\partial_t \rho_i)_{LC-} = & -\frac{1}{2} \cdot \rho_i \cdot e^{-\frac{\tilde{\rho}_{(i+1)}}{\tilde{\rho}_i}} \cdot \int_{v_{min}}^{v} \int_{v_{min}}^{v_{max}} \rho_i(u, v_0, x, t) \beta(u, v) dv_0 du \\
& -\frac{1}{2} \cdot \rho_i \cdot e^{-\frac{\tilde{\rho}_{(i-1)}}{\tilde{\rho}_i}} \cdot \int_{v_{min}}^{v} \int_{v_{min}}^{v_{max}} \rho_i(u, v_0, x, t) \beta(u, v) dv_0 du
\end{aligned} \tag{36}
$$

$$
\begin{aligned}
(\partial_t \rho_i)_{LC+} = & \frac{1}{2} \cdot \rho_{(i+1)} \cdot e^{-\frac{\tilde{\rho}_i}{\tilde{\rho}_{(i+1)}}} \cdot \int_{v_{min}}^{v} \int_{v_{min}}^{v_{max}} \rho_{(i+1)}(u, v_0, x, t) \beta(u, v) dv_0 du + \\
& \frac{1}{2} \cdot \rho_{(i-1)} \cdot e^{-\frac{\tilde{\rho}_i}{\tilde{\rho}_{(i-1)}}} \cdot \int_{v_{min}}^{v} \int_{v_{min}}^{v_{max}} \rho_{(i-1)}(u, v_0, x, t) \beta(u, v) dv_0 du
\end{aligned} \tag{37}
$$

Traffic Safety

In our microscopic model, we have already computed the the microscopic accident rate concerning two cars. To apply the result to the macroscopic case, we only need to do some weighted multiple integrations on the microscopic accident-rate-functions.

- Accident coefficient for the overtaking in lane i at distance x is

$$
\begin{aligned}
d^i_{overtake} = & \int_{v_{min}}^{v_{max}} \int_{v_{min}}^{v_{max}} \int_{v_{min}}^{v_2} \int_{v_{min}}^{v_2} \rho_i(u_1, v_1, x, t) \times \rho_{i-1}(u_2, v_2, x, t) \times \\
& \left(e^{t \frac{|u_2 - u_1|}{l_{notice}}} - 1 \right) du_1 du_2 dv_1 dv_2
\end{aligned} \tag{38}
$$

[2]For simplicity, we do not write explicitly the boundary terms. For boundary lane($i.e.$the right most&the left most),there is only one term and the factor $\frac{1}{2}$ would become 1 since they only have one choice to overtake. Moreover, for lanes next to the boundary lane, the $LC+$ term from the boundary lane would have a factor 1 instead of $\frac{1}{2}$, by the law of reservation.

- Accident coefficient for lane changing in lane i at time t is therefore

$$d_{LC}^i = \int_{v_{min}}^{v_{max}} \int_{v_{min}}^{v_{max}} \int_{v_{min}}^{v_1} \int_{v_{min}}^{v_2} \int_0^L \int_{x_1}^L \rho_i(u_1, v_2, x_1, t) \times$$
$$(\partial_t \rho_i)_{LC^-}(u_2, v_2, x_2, t) \times d_{LC1}(v_1, v_2, x_2 - x_1) dx_2 dx_1 du_2 du_1 dv_2 dv_1$$
$$+ \int_{v_{min}}^{v_{max}} \int_{v_{min}}^{v_{max}} \int_{v_{min}}^{v_1} \int_{v_{min}}^{v_2} \int_0^L \int_0^{x_1} \rho_i(u_1, v_2, x_1, t) \times$$
$$(\partial_t \rho_i)_{LC^-}(u_2, v_2, x_2, t) \times d_{LC2}(v_1, v_2, x_2 - x_1) dx_2 dx_1 du_2 du_1 dv_2 dv_1 \quad (39)$$

- Accident coefficient for acceleration on a same lane is

$$d_{rear}^i = \int_0^L \int_0^L \int_{v_{min}}^{v_{max}} \int_{v_{min}}^{v_{max}} \int_{v_{min}}^{v_2} \int_{v_{min}}^{v_2} \rho_i(u_1, v_1, x_1, t) \times \rho_i(u_2, v_2, x_2, t) \times$$
$$d_{rear}(v_1, v_2, |x_1 - x_2|) du_1 du_2 dv_1 dv_2 dx_1 dx_2 \quad (40)$$

4 Numerical Results and Analysis

Our mathematical models have given an elaborate description of the traffic. However, we can tell that the equations and formulae are very complicated and is almost to find an explicit solution. Thus we have to turn to numerical methods. We have to divide the continuous phase-space into many small grids to solve the partial differential equation.

4.1 Parameters

- Each grid is taken as a $1m \times 1s$ square in the time-space.
- We simulate the case when there are 4 lines.
- The left-most to the right-most 4 lanes are orderly labelled as 1,2,3,4
- The total length of road simulated is $12000m$ long.
- The total length of simulation is $20000s \approx 5.5 hours$.
- Maximum speed and minimum speed is respectively $33m/s$ and $17m/s$.
- The maximum acceleration and maximum deceleration is respectively $5m/s^2$ and $7m/s^2$.
- The maximum speeds of cars are initiated as approximately normal distribution $N(25, 4)$.
- When simulating traffic under the rule, the entering cars are all at the right-most lanes at any time.
- When simulating traffic without the rule, the entering cars have the equal probability to enter the road from any lane at time 0.
- The entering flow is a changeable parameter.

4.2 Direct Output: Traffic Density

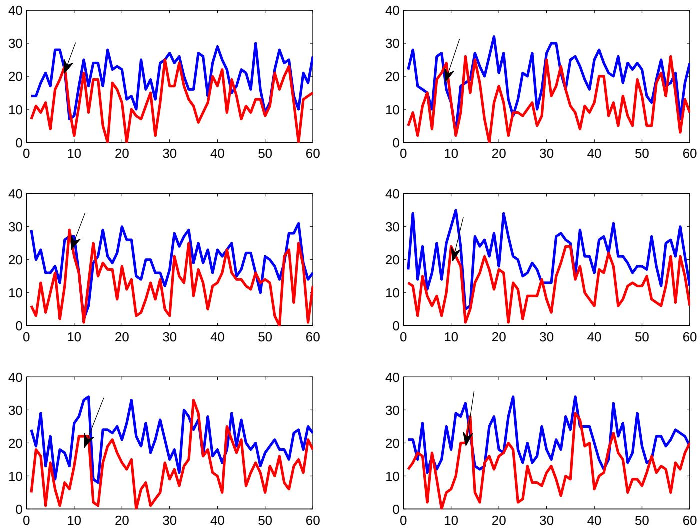

Figure 4: Density Curve

The figure shows the density curve of lane 4 (blue) and lane 3 (red). Since the density of each grid is too small to give a clear view of the entire road. We divide the road into 60 segments and compute the total density of each segment. And the time difference between each graph is $10s$. Although not very obvious, we can tell the density has a wave shape, going high and low periodically. Besides, if we see more carefully, we can realize the traffic wave is moving right just like a physical wave. Looking at the black arrow in the figure, it points at one peak point of density. As time passes, we can see the peak point is moving to the right.

When we see a wave shape, our first reaction is always to do **Fourier Analysis**. The following figure shows the result of $FastFourierTransform$(FFT) of the density curve. As we do the Fourier transform at each time point, we can see that the high frequency terms behave quite unstably. However, for the first three amplitude peaks, it stays at the same frequency and same amplitude as time passes. Especially the first peak, which is also the largest peak behaves very stably. This fact verifies our observation of periodicity.

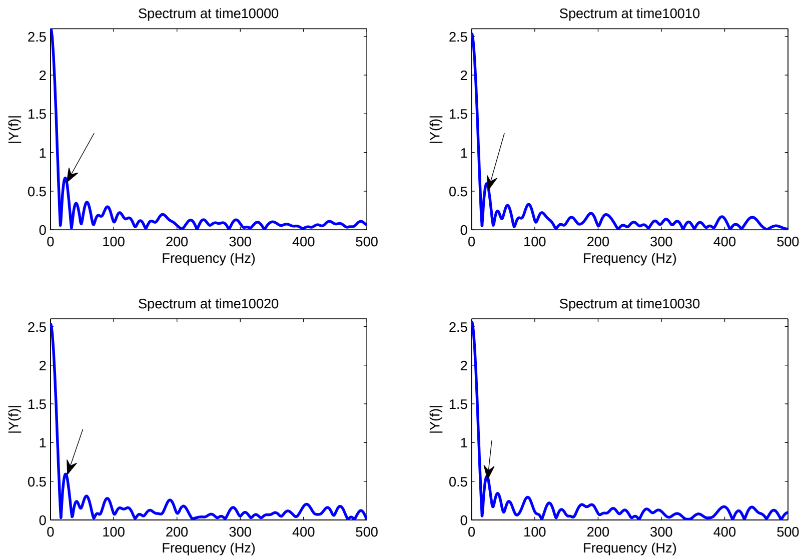

Figure 5: Spectrum of the Density Curve

4.3 Cumulative Density

Although we have seen the density curve on each line, we are more interested in the cumulative density of each line. That is which lane is more crowded and which lane is more vacant. By a simple integral operation (sum in numerical simulation), we can obtain the following result.

Table 2: Average Cumulative Density

Average Cumulative Density	Lane 1	Lane 2	Lane 3	Lane 4
Under the Rule	5.5354	32.5088	87.1061	130.3363
Without the Rule	62.5940	70.4822	68.7449	58.2846

From the table above, we can tell that under the *Keep-Right-Except-To-Pass* rule, from the right-most lane to the left-most lane, the cumulative density increases significantly. This makes sense because cars drive on left lanes only when overtaking. For instance, a car runs on lane 1 only if there are cars ahead on lane 2,3,4 and they are all slower. Therefore the cumulative density of lane 1 is very small.

On the other hand, when there is no *Keep-Right-Except-To-Pass* rule, the cumulative density of four lanes are very close. One can tell that the middle two lanes are a little bit more crowded. This can be nicely explained by a simple binomial probability model. Here we only give the major idea. For cars on lane 1 heading for overtaking, they can only choose to overtake on lane 2. However, for cars on lane 2, they have probability $\frac{1}{2}$ to choose to overtake on lane 3 and probability $\frac{1}{2}$ to choose to overtake on lane 1. And cars would stay on its overtaking lane. As a consequence, more cars would be on the middle lanes than the boundary lanes.

In summary, we know that the *Keep-Right-Except-To-Pass* rule actually require car to always fulfill the right lanes first and drive on the left lanes only when right lanes are occupied, either to overtake because there is a slower car occupying your space or to wait to return back to the right lane because there is not enough space.

Impact 1 *The rule makes the traffic to use the right lanes first and then use the left lanes when right lanes are occupied.*

4.4 Speed Distributions on Different Lanes

The *Keep-Right-Except-To-Pass* law requires that a car uses the left line only when overtaking. So it is intuitively natural that the average speed on each lane would be different. To be precise, we would always expect the average speed on the right lane higher than the left lane because the cars on the left lane are all to overtake cars on the right lane. At every second of the simulation, we record the *equivalent velocity* of each grid as

$$v_{equi}(x,t) = v \cdot \int_{v_{min}}^{v_{max}} \rho_i(v, v_0, x, t) dv_0 \tag{41}$$

Then after the simulation, we do a histogram for recorded velocities on the lane. Following are the results.

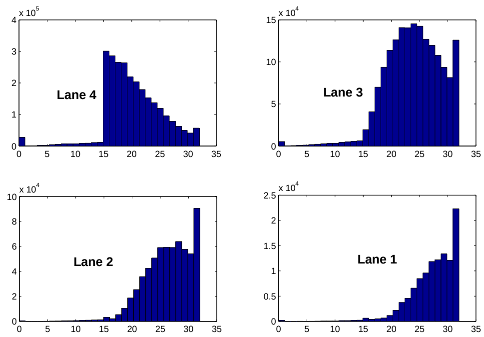

Figure 6: Velocities

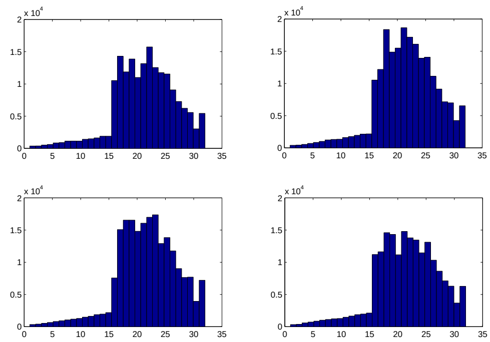

Figure 7: Velocities-reference group

To our expectation, the speed distribution is moving to the right as the lane changes from lane 4 to lane 1. One important fact to point out here is that for lane 1,2,3, the there is a very tall bin at the right most. This is due to the upper speed limit. One can easily imagine that if there's no upper speed limit, there would be more short bins on the high speed domain and the histogram would look very smooth.
The right histogram is of the reference group where cars can overtake both left lane and right lane and do not have to return to the original lane. Again, to our expectation, the speed distribution on the four lanes are almost the same. This is because without the *Keep-Right-Except-To-Pass* rule, cars are free to drive on any lane.

Impact 2 *The Keep-Right-Except-To-Pass rule 'pushes' fast cars to the left lanes and slow cars to the right lanes.*

We can also do some statistical summary on the different maximum speeds of cars on different lanes. The output is very similar. When there is the rule, the left lane always has more cars with bigger maximum speeds than that of the left lane. Here we omit the details. The following two tables gives the summary of the output.

Table 3: Summary of Velocties

Average Velocity	Lane 1	Lane 2	Lane 3	Lane 4
Under the Rule	27.2683	26.1572	23.8577	20.3733
Without the Rule	21.3662	21.5602	21.3978	21.2079

Table 4: Summary of Maximum Velocities

Average Maximum Velocity	Lane 1	Lane 2	Lane 3	Lane 4
Under the Rule	29.0716	27.6923	25.6454	22.3744
Without the Rule	22.5550	23.5689	23.1076	22.1483

4.5 Low & Heavy Traffic

Before we give our numerical result, we can first do some reasoning on how would the rule perform differently in light and heavy traffic. We have proposed two impacts of law, which can be briefly summarized as these two principles: first right, then left; slow right, fast left. As we can see, the rule is actually making the road traffic more organized. Everyday experience tells us that **order** is of great importance when crowded but could be harmful when there is enough free space. To be more precise, when in light traffic, there are very few cars and the rule requires the cars to drive on the right-most. Then the consequence is that most of the space of the other lanes are wasted. They are empty and can be used for driving but just not allowed.

Let us do a simple computation here. When there are few cars on the road, we assume that a car would never encounter two cars at the same time. Then for a car with speed v_0, since all cars are running on the right-most lane, it would have to overtake all cars with speed$< v_0$ he passes. However, if there isn't the rule, all lanes are made use of. Then the cars running slower than you are uniformly distributed on the four lanes. Thus you would only have to overtake approximately $\frac{1}{4}$ of cars with speed$< v_0$.

But when there is heavy traffic, it is a different story. Under such circumstances, all lanes are made full use of and are fulfilled with vehicles. This time the rule performs positively. It is well-known and deeply studied that a slow leading car would reduce the speed of many cars driving behind it. So we would the slow cars drive on the same lane so such negative interaction would be minimized. In summary, we can summarize this as **order and freedom are the two sides of a coin.**.

The numerical result verifies our reasoning. In this simulation, we let the boundary term $\rho_i(0, v_0, 0, t)$ to increase constantly thus the traffic pressure is getting higher and higher. Then we can see that when the traffic pressure is low, the without-rule road has a higher traffic flow. But when the traffic pressure gets big, the under-rule road shows its advantage. We cannot tell exactly at how much traffic pressure the *Keep-Right-Except-To-Pass* rule begins to work. But we can tell the general trend.

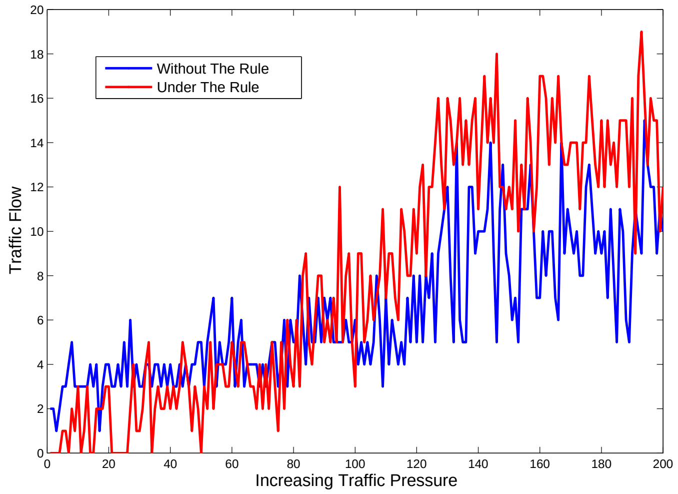

Figure 8: Traffic Flow in Light & Heavy Pressure

Impact 3 *The Keep-Right-Except-To-Pass rule promotes traffic flow when there is heavy traffic and decreases traffic flow when there is light traffic.*

The rule's advantage does not only include promoting traffic flow when traffic is heavy. It also largely decreases the accident coefficient. From the figure we can tell that when traffic pressure is light (< 100,approximately), the accident coefficient with and without the rule is very close. However, as the pressure gets heavy, the accident coefficient without the rule increases much more fast than that with the rule. This is because when there is no *Keep-Right-Except-To-Pass* rule, overtaking would happen more frequently **since fast and slow cars are not divided into different lanes** . Moreover, the velocity difference between the leading car and the following car is also larger for the same reason, which as well add to accident coefficient.

Impact 4 *The rule largely decreases the accident coefficient when traffic is heavy.*

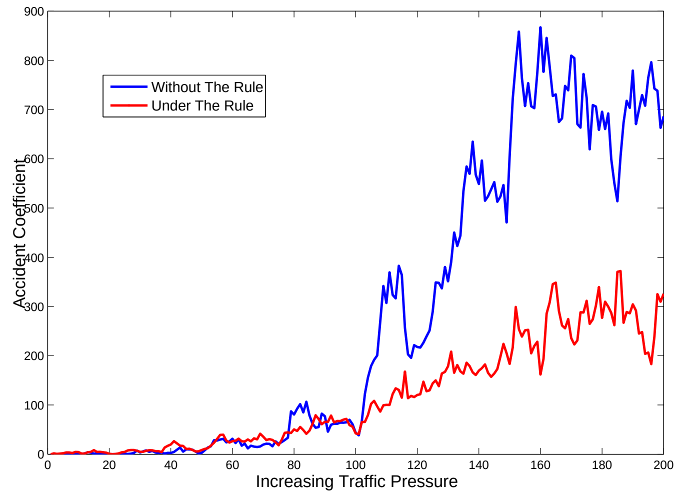

Figure 9: Accident Coefficient in Light & Heavy Pressure

4.6 Tradeoff Between Flow and Safety

As discussed in our micro-&macro-scopic model, we use the accident coefficient, which can be regarded as the probability of an accident. This index as well as the traffic flow are determined by many parameter. To analyze their relationship, we constantly change the parameters and at the same time record the Accident Coefficient and Traffic Flow at each time unit. Then we obtain the following scatter plot.

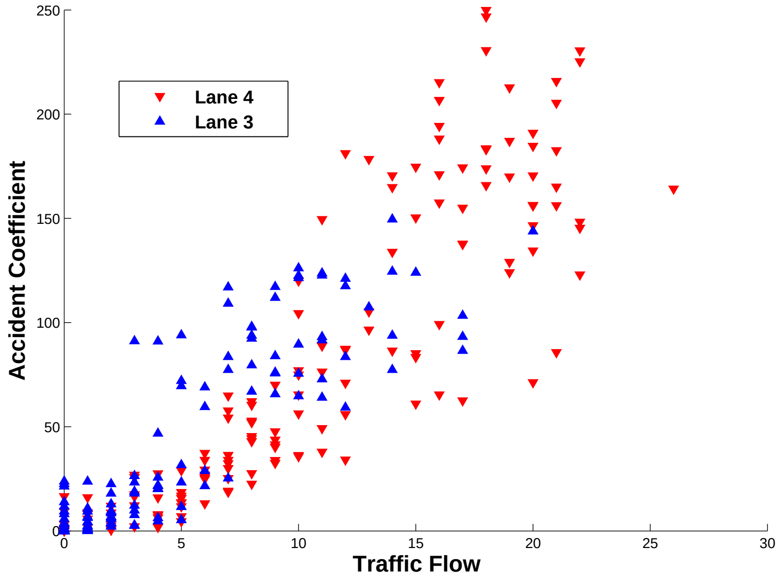

Figure 10: Flow & Safety

As we can see the Accident Coefficient has a strong negative correlation with the Traffic Flow. That is to say, higher the traffic flow, more likely an accident would happen. We further compute their correlation factor to verify our observation.

$$cor(Acc, Flow) = \frac{0.2851}{\sqrt{2.8209}\sqrt{0.0347}} = 0.9113 \tag{42}$$

Then correlation between these two index are as high as 0.9113. This shows they are strongly correlated.

4.7 Under-&Over-posted Speed Limits

Most freeways have a speed limit. It can be regarded as a controlling parameter to adjust the traffic flow. In our model, we assume the wanted maximum velocities of cars are normally distributed as $N(25, 5^2)$. We simulate the case when speed limits equals to 21 to 29. In each case, we record the average traffic flow and average accident coefficient of the road. Following is the output. We can see that as speed limits goes up, the accident coefficient and traffic flow both increases. Thus here is again a tradeoff between flow and safety. If the speed limit is under-posted, the safety index would be high but the traffic flow would be small. On the contrary, if the speed limit is over-posted, the road would have high accident coefficient as well as high flow. As too high accident coefficient and too low traffic low are both wanted, we need a balance between safety and efficiency. We define the following factor as the aggregative index of the road.

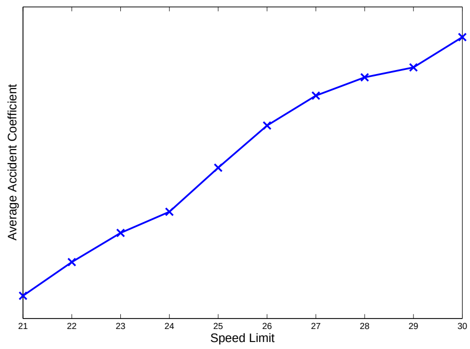

Figure 11: Accident Coefficient

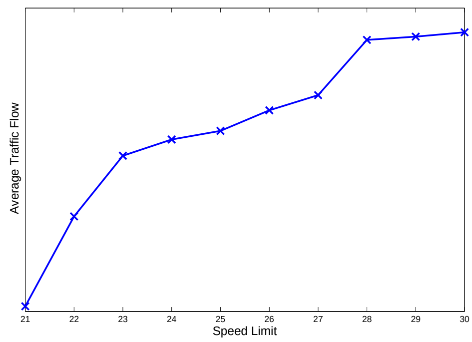

Figure 12: Traffic Flow

Definition 5 *Aggregative index*$\lambda = e^{-d} \cdot f^2$ *where d is the accident coefficient and f is the traffic flow.*

When we plot the index we can tell that the aggregative index goes up at first and then goes down. This implies that a modest value of speed limit would give a best road condition.

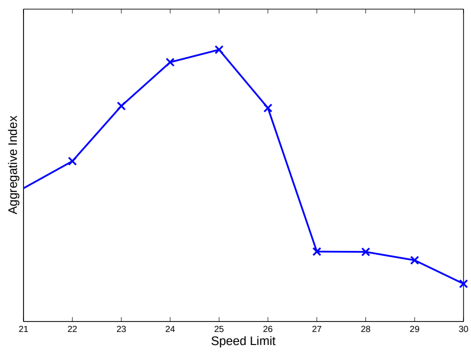

Figure 13: Aggregative Index

Note that how the aggregative index should be defined is very subjective and having no right answer. We just propose one possibility to make the issue clear.

4.8 Number of Lanes

We would like to find the relation between number of lanes and the flow/accident/aggregate coefficient. The increase of lanes will increase the vehicle flow, thus adding the probability of the occurrence of a single accident. The accident coefficient increases as well as the vehicle flow.
When we carefully examine the data generated, we can see that the aggregative coefficient goes down rapidly while the quantity of flow increases. This is because larger flow results in more risks of car crashing. We can also see that although the aggregate coefficient for 4 lanes is smaller than that of 3 lanes at the beginning, it is greater when further flow pressure comes in.
One important remark here is that all the coefficients are average values of each lane. **This means a road with 4 lanes performs better than 2 roads each with 2 lanes.**This '1 + 1 > 2' effect shows that under the rule, more lanes really help improve the total

condition.

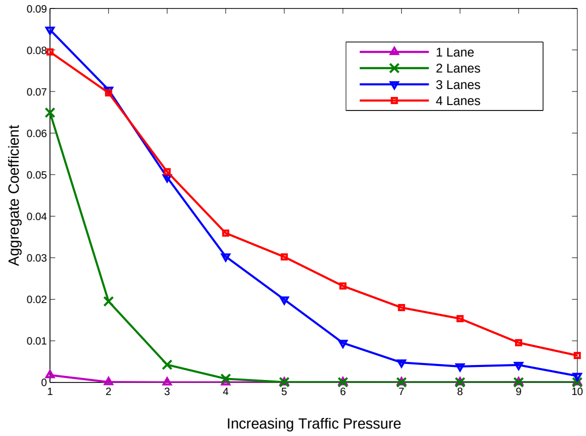

Figure 14: Aggregative Coefficient with Different Number of Lanes

5 Alternative Rules

In this section, we consider and analyze alternative lane-changing rules.

5.1 No Rule At All

In previous discussion, to analyze the performance of *Keep-Right-Except-To-Pass* rule, we propose a reference group. In the reference group, there is no strict rule about lane-changing. So the cars would obey the so-called *lazy rule* that a car would only change lane to overtake. Previous output has already illustrated then characteristics of this rule. Here we give a summary.

Performance of No Rule At All

- When the traffic is light, it frees the cars to make use of the whole road resources thus obtain a higher traffic flow.
- When the traffic is heavy, the accident coefficient is relatively very high due to frequent lane-changing, overtaking and speed shifting.
- When the traffic is heavy, the traffic flow is relatively very low due to disorder.
- Surely, the drivers feel most free under this rule.

5.2 Variable Speed Range

Keep-Right-Except-To-Pass performances very well when traffic is heavy but is of low efficiency when traffic is light. These are facts we see. The underlying advantage and disadvantage are:

- **Advantage:** divide cars with different speeds into different lanes.
- **Disadvantage:** not always make full use of the road space.

Our new rule is simply motivated to keep the advantage and avoid the disadvantage of the *Keep-Right-Except-To-Pass* rule. This rule is actually alreadyu common in many countries and regions.

A New Rule:

- Assign every lane a different speed range. The speed range of the left lane is higher than that of the right lane.
- Cars are required to drive at the lane whose speed range contains its current speed. We call it the car's should-be-lane.
- Overtake on the left.
- If a car's should-be-lane is fully occupied, it runs on the lane left to the should-be-lane. We say it be the second should-be-lane. Similarly we have third should-be-lane.

When traffic is light, it avoids wasting road space since all four lanes are having cars with speed in its speed range on it. When traffic is heavy, it would not lose order like No Rule At All since the vehicles are divided by its velocities. Therefore this rule is expected to perform better. The following figure shows the numerical result.

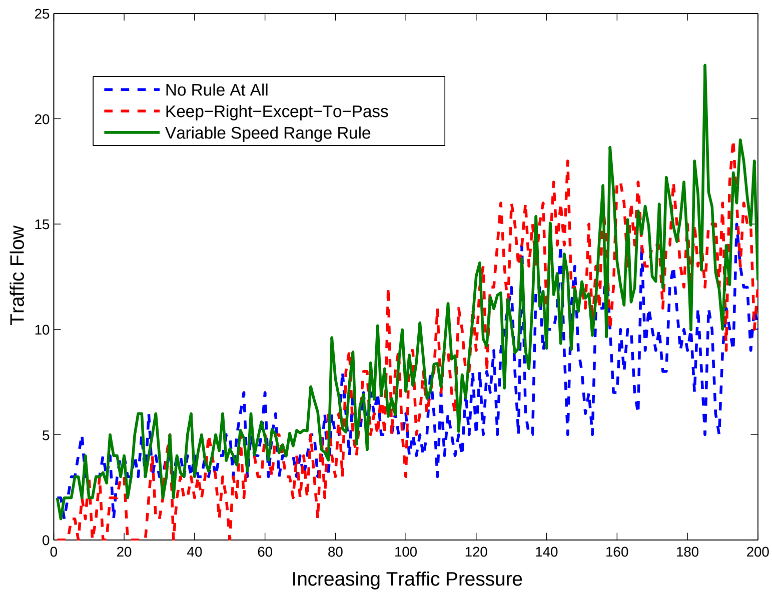

Figure 15: Comparison of the Three Rules: Traffic Flow

From the figure we can see that this new rule successfully promotes flow in both light and heavy traffic. It is better than *Keep-Right-To-Pass* in light traffic and better than *No-Rule-At-All* in heavy traffic. Although outperforming two previous rules in promoting flow, this rule does not do the best when it comes to safety.
As the figure shows, the accident coefficient of this new rule increases faster than the *Keep-Right-To-Pass* rule as traffic get heavy. This is mainly because in crowded roads, the cars cannot always drive on its should-be-lane thus overtakes quite frequently switching between its second or third should-be-lane and its should-be-lane.

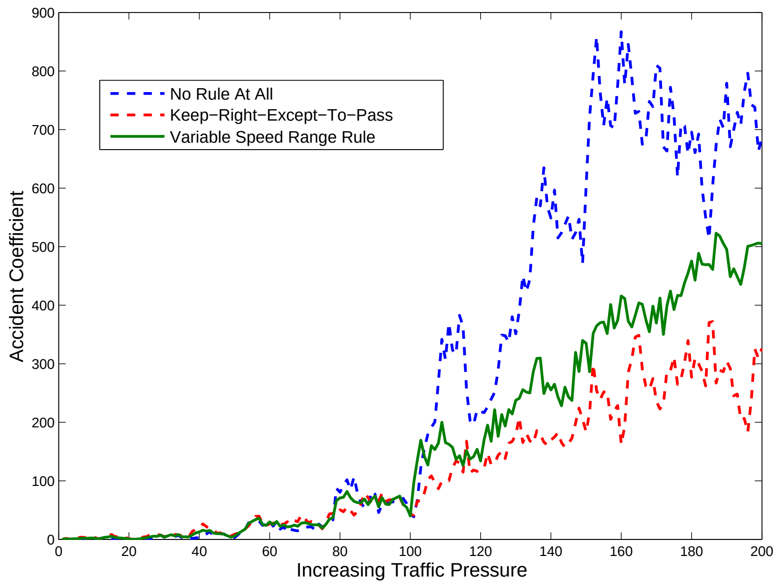

Figure 16: Comparison of the Three Rules: Accident Coefficient

Performance of Variable Speed Range Rule

- It promotes traffic flow regardless the traffic pressure.
- Its accident coefficients in heavy traffic are between that of No-Rule-At-All and Keep-Right-Except-To-Pass.
- It avoids **serious** traffic accident since the velocity difference would not be large between two cars.
- This rule is **flexible**. Because a control system can change the required speed range of every lane as road condition changes.

5.3 Restricted Distance Between Vehicles

In introducing new rules stated above, we focus more on increasing traffic flow to improve traffic capacity. In order to improve the overall performance of a highway, we also think about a rule to improve the safety. Distance between two cars is a very important factor that affects both safety and efficiency. Previous analysis assume the car distance relying on the judgement of an individual driver as a function of the speeds of the leading car and following car. However, we propose a new rule here that requires a minimum car distance.

- Distances between any adjacent cars in the same lane cannot be lower than a distance limit G.
- Drivers still obey *Keep-Right-Except-To-Pass* rule.

In order to find the best value of this distance limit, we have done a numerical experiment. With all the other parameters fixed, we change G to see the result which in shown in the following figures. We can see from the figures that the accident coefficient decreases as the distance limit increases which is a intuitive result, and the average traffic flow decrease

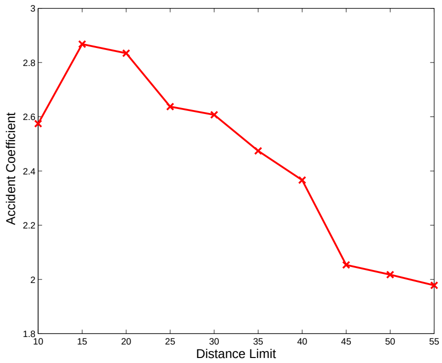

Figure 17: Accident Coefficient

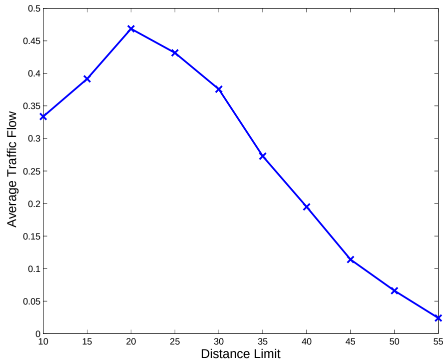

Figure 18: Average Traffic Flow

when the distance limit increases which is also intuitive except when the distance limit is very small. So we can see that a too small distance limit may not increase the traffic flow efficiently.Theses two types of data again generate relation between the aggregative coefficient we defined before and flow. We can see that there is an optimal value of distance limit to get the best performance of a highway. (And this value exists, here the value is 25)

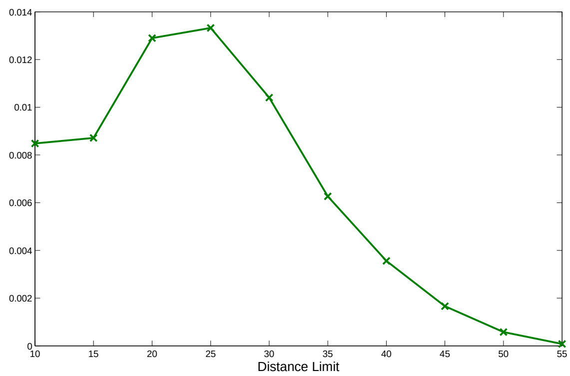

Figure 19: Aggregative Coefficient

Performance of Rule with Distance Limit

- The rule with distance limit may help the highway to reach its optimal performance if the value of the distance limit is appropriate.

6 Switched Orientation

According to the highway code No. 137-138 in UK, the rule requires drivers to drive in the left-most lane of a multi-lane highway unless they are passing another vehicle, and other lanes on the right hand side are considered to be passing lanes. So the lane arrangement of left-hand traffic are exactly the **mirror image** of that of right-hand traffic. Moreover, most countries or regions with right-hand traffic rules use left-hand driving cars while most with left-hand traffic rules use right-hand drive cars, except in Hongkong, for instance. As a result, the different view field between left- and right-hand drive cars are also mirror

images under the circumstances caused by corresponding traffic rules.
Both the traffic flow and the accident coefficient will not change up to the principle of this model. Just imagine the system is operating in the world that is the mirror image of our world will suffice. The only change that may be taken into account are the biological (handedness of human beings) or geological difference(the latitude).
To make it more clear, a driver in a left-hand drive car has a better view field of his left-hand side so it is easier for him to change to the left lane which is the passing lane. At the same time, a driver in a right-hand drive car has a better view on his right, and the lane on his right is also the passing lane. Consequently, the human body and the big environment, the earth, are other things that may cause asymmetry between left- and right-hand traffic. This is the biological effect. Because most people are right-handed, driving a right-hand drive car is different from driving a left-hand drive car for human beings. And, vehicles at different places of the world will experience different **Coriolis force** on the rotating earth. They are two different factors in the left- and right-hand traffic rules. We now analyse these two factors.

- **Coriolis Force**
  First, we analyse the influence of the Coriolis force. We estimate the magnitude of Coriolis acceleration that is exerted on a moving car on a highway at mid-latitude places. The Coriolis acceleration is given by $a_c = 2 \cdot \omega \times v = 2|\omega||v| \sin\theta$ where ω is the angular velocity of the rotating frame and v is the velocity of the moving object in that frame and θ is the angle included between ω and θ. The angular velocity of the earth is given by $\omega = \frac{2\pi}{24 \cdot 3600} \approx 7.27 \times 10^{-5}$. Assume the latitude is 45° and the highway is northbound or southbound, so $\theta = \frac{\pi}{4}$, and assume the velocity of a car is 30 m/s. We can estimate the magnitude of the Coriolis acceleration:

$$a_c = 2|\omega||v| \sin\theta = 2 \times 7.27 \times 10^{-5} \times 30 \times \frac{\sqrt{2}}{2} \approx 0.003 m/s^2 \tag{43}$$

  As we can see, the Coriolis acceleration is only 0.01% percent of the velocity. Moreover, the frictional force of the road can almost completely counteract the Coriolis force. Thus it is of no significance in real-life driving.

- **Asymmetry of human body**
  There is an interesting biological phenomenon that a driver are more likely to turn left to avoid collision when there is an obstacle in front of his car. The reason is that the heart is in the left part of our body, so human tend to protect his left body. However, this phenomenon may have influence on the traffic only when there is a very emergency condition, for example, when emergent braking is useless. This fact may result in a higher accident coefficient for countries driving on the left of the automobile. But it can hardly make a significant difference under normal circumstances.

Consequently, there can hardly be noticeable other differences between the traffic or traffic rules in country with left-and right hand traffic rule except that they are mirror images of each other.

7 Intelligent Traffic System

With the enormous development in the field of information and control technology, intelligent traffic system nowadays is approaching near to our real life. There are different types of intelligent traffic system with different levels of intelligence and all of them have features in common as well as their own. So to get a brief view over the intelligent traffic system with lane-changing rule, we analyze the general and common features of these systems instead of any one specific system.

7.1 Main Features of Intelligent System

We assume the following three are the most important features of intelligent traffic system as a summary of [5]:

- **Fast Reaction:** A driver takes some time to react to a certain event and make an decision. But the reaction time of an intelligent system can be neglected.
- **Overall Information:**A driver can only see the cars and road condition near him thus can only obtain local information. But through the intelligent system, a car has access to the information of the entire road.
- **Synergic Control:** A driver can only control his own car. But the intelligent system can control more than one car simultaneously.

7.2 Shorter Safety Distance

Now we consider how the first feature **Fast Reaction** would change our analysis. If one studies our micro-&macro-scopic model carefully, he or she would realize that the **Safety Distance** is one of the core parameters in the model. Almost every term of the main formula is related to safety distance. We recall, twice again, the formula of safety distance.

$$L_{safe} = (v_0 - v_1) \cdot t_1 + \frac{(v_0 - v_1)^2}{2d_{max}} \tag{44}$$

t_1 is the reaction time of a driver and the second term is determined by the mechanical parameter of the car (d_{max}). So under the control of intelligent system, the reaction time can be ignored thus the formula of safety distance is modified to be:

$$L_{safe} = \frac{(v_0 - v_1)^2}{2d_{max}} \tag{45}$$

7.3 Variable Speed Range Rule Gains More Advantage!

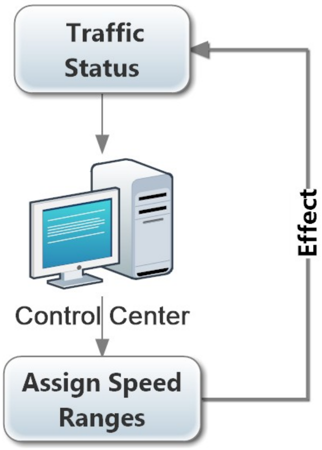

Figure 20: System Controlling the Speed Range

As listed previously, one important factor of our proposed variable speed range rule is that it is **flexible**. A trivial approach is that one can change the required speed range of each lane due to different hours in a day. For example, rush hours and midnight makes a big difference. But under the help of intelligent traffic system, this would be very powerful. The system has access to the traffic information of the entire road thus is capable to find the timely optimal speed range for each lane. Even more, the system can assign different speed range for one lane at different positions of the entire road. Under this intelligent control, the drawback of inflexibility of the Variable Speed Range rule is removed.

8 Strength & Weakness

Strength:

- In our microscopic model, we have considered several different cases in which traffic accidents may happen.
- The accidents coefficients are dimensionless, so they are invariant under the change of magnitudes of data that we use in our program.
- The accident coefficients are easy to compute in the real case, as the distance and velocity of adjacent vehicles can be easily measured.
- The gas-like equation set model is of great mathematical integrity.
- A hierarchy of models successfully settles the high complexity of real-life traffic.
- The model can run stably even when parameters are largely changed.
- We can use different combination of methods for different problems at different levels in traffic. And the application of our model is not restricted to lane-changing rules.
- A large number of factors are taken into account thus various different analysis can be done.

Weakness:

- As we have defined three accident coefficients $d^i_{overtake}, d^i_{LC}$ and d^i_{rear} in three different way in three cases, it is difficult to balance the importance among them to measure the overall accident coefficient.
- Since a traffic accident may be caused by a mixture of factors, it is not so scientific to measure the safety in different cases separately.
- The partial differential equations involved in this model include both differential operators and integral operators, so they are extremely difficult to solve mathematically.
- Solving numerically is of very high computation complexity. Usually it takes half an hour to get one solution with programs in MATLAB.
- We ignore the existence of some special situations like violators and sudden braking, which could affect the traffic largely.

References

[1] Hoogendoorn, Serge P., and Piet HL Bovy. "State-of-the-art of vehicular traffic flow modelling." Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering 215.4 (2001): 283-303.

[2] Laval, Jorge A., and Carlos F. Daganzo. "Lane-changing in traffic streams." Transportation Research Part B: Methodological 40.3 (2006): 251-264.

[3] Uno N, Iida Y, Itsubo S, et al. A microscopic analysis of traffic conflict caused by lane-changing vehicle at weaving section[C] Proceedings of The 13th Mini-Euro Conference" Handling Uncertainty in Transportation Analysis of Traffic and Transportation Systems. 2002: 143-148.

[4] Prigogine, Ilya. "A Boltzmann-like approach to the statistical theory of traffic flow." (1900).

[5] Wikipedia contributors, 'Intelligent transportation system', Wikipedia, The Free Encyclopedia, 16 January 2014, 16:28 UTC, `<http://en.wikipedia.org/w/index.php?title=Intelligent_transportation_system&oldid=590985221>` [accessed 10 February 2014]